Trajectory Generation for Constant Velocity Target Motion Estimation Using Monocular Vision

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Abstract - The performance of monocular vision based target tracking is a strong function of camera motion. Without motion, the target estimation problem is unsolvable. By designing the camera path, the best possible estimator performance can be achieved. This paper describes a trajectory design method based on the predicted target state error covariance. This method uses a pyramid, breadth-first search algorithm to generate real-time paths that achieve a minimum uncertainty bound in fixed time or a desired uncertainty bound in minimum time.

1. Introduction

Monocular vision based target tracking is an important estimation method for autonomous field robots. Computer vision is an information-rich sensor that provides multiple capabilities including object detection, identification, and tracking. Single-camera solutions are useful for many vision applications. For example, micro air vehicles that provide aerial surveillance have size constraints that limit the available payload. The motion of a target object can be estimated from a single image track if the camera motion is known.

One important issue is the dependence of the target estimator performance on the camera motion. It is well known that for monocular vision based tracking the object motion is unobservable at any instant in time and that only certain camera paths yield solutions [1, 10]. Furthermore, favorable camera motions can be generated that achieve good estimator performance.

The trajectory design problem is not new and has been studied extensively for the related problem of passive sonar applications[2-8]. In the sonar literature, the estimation problem is referred to as bearings-only tracking or target motion analysis. The object is generally referred to as the target and the bearing sensor - the camera in this case - is referred to as the observer.

The concept of designing the observer trajectory in order to optimize estimator performance was first introduced by Hammel et al.[2]. They addressed the problem of trajectory design for a stationary target by using the Fisher Information Matrix (FIM) to describe the performance of an ideal estimator and by designing a numerical algorithm that minimized its determinant. They also derived lower bounds which could be used to generate observer paths analytically.

Oshman et al.[3] extended the stationary target problem by incorporating observer motion constraints. For the passive sonar application these constraints come mainly from hostility functions describing the operational environment.

Trajectory design for a moving target was studied by LeCadre and Jauffret [5]. They linearized the target motion analysis problem by transforming the measurement equations into a linear pseudomeasurement. They also used a discrete time formulation in order to reduce the analysis to multilinear algebra. They showed that the optimal path can be calculated for a target with arbitrary maneuvers under certain assumptions.

The problem addressed in this paper is the design of observer trajectories for monocular vision based tracking. Trajectories are desired that can be generated in real-time to minimize the estimate uncertainty in a given time or to minimize the time needed to achieve a specified error bound. Our approach is to extend the results of the sonar-based literature by addressing three separate issues.

First, the restricted camera field of view must be incorporated. In contrast to sonar systems that typically have a full 360 degree field of view, the computer vision system is constrained such that a measurement can only be taken if the camera is pointed towards the target. Some cameras cannot be moved independently from their heading and therefore cannot always remain pointed at the target (for example, a camera fixed to a ground rover). Hence, the trajectory design method must allow for motion that brings the target out of the camera field of view.

A second extension is needed in order to enable solutions of both the desired problems. Previous results have only minimized the uncertainty of the target estimate in given time. Here, the alternate problem is also included - minimizing the time taken to achieve a specified estimate uncertainty.

The final extension enables fast generation of the observer trajectories. The search space is too large to enable fully optimal solutions in reasonable time so a new, real-time strategy must be developed [8].

In order to accomplish these extensions a new optimization method is presented. A breadth-first search algorithm is used to generate a set of candidate trajectories that satisfy all constraints imposed on the system. A pyramid iteration scheme is used to calculate fast, sub-optimal paths. The optimization cost is defined using the predicted target error covariance matrix. Minimizing the determinant of this matrix is equivalent to maximizing the mutual information between the observer trajectory and the final target state [7].

The following sections present the new trajectory design method. The monocular vision based tracking estimator is first described in order to show its dependence on the observer motion. The details of the new trajectory design method are then presented, including the new cost function and new optimization method. A typical tracking scenario is simulated and the results show the successful estimation of target motion using the trajectory design method.

2. Monocular Vision Based Motion Estimation

Although the methods presented in this paper apply to the full three dimensional problem, only the 2-D planar case is presented. Hammel and Aidala[4] show that for the bearings-only tracking problem the 3-D results parallel the 2-D ones.

The geometry of the target motion estimation problem is shown in Figure 1. The target state is described by its position and velocity. The estimation state vector is

$$X = X_{target} = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^T \qquad (1)$$

For a constant velocity target, the state update equation is

$$X_{target}[k+1] = \Phi \cdot X_{target}[k] + \Gamma \cdot w[k] \qquad (2)$$

$$\Phi = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} 0.5\Delta t^2 & 0 \\ 0 & 0.5\Delta t^2 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \qquad (3)$$

where $w[k]$ represents small accelerations that are treated as zero-mean Gaussian white process noise with covariance matrix $R_w = -E[ww^T] = \sigma_w \cdot I$.

This work assumes the motion of the observer is known. In this case, the motion of the target can be determined from a single point feature.

The computer vision measurement is the projection of the relative target position onto the image plane. The vision measurement is

$$z[k] = h(X[k]) = f \cdot (x_s / y_s) + v \qquad (4)$$

$$\begin{aligned} X_s &= \begin{bmatrix} T_{w2s} & 0 \\ 0 & T_{w2s} \end{bmatrix} \cdot (X_{target} - X_{observer}) \\ &= \begin{bmatrix} x_s & y_s & \dot{x}_s & \dot{y}_s \end{bmatrix}^T \end{aligned} \qquad (5)$$

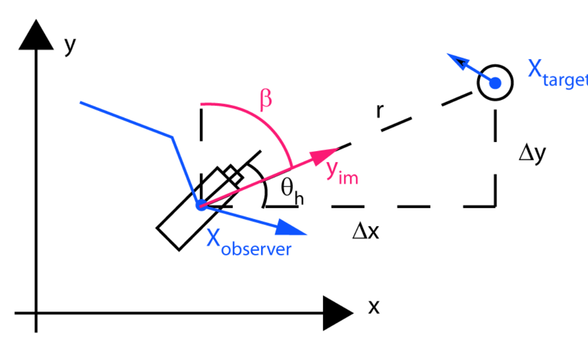

Figure 1. Problem geometry

$$T_{w2s} = \begin{bmatrix} \cos(\theta_h) & -\sin(\theta_h) \\ \sin(\theta_h) & \cos(\theta_h) \end{bmatrix} \quad (6)$$

where $v[k]$ is zero-mean Gaussian white measurement noise with covariance matrix $R_v = E[vv^T] = \sigma_v$. $X_{observer}$ is the known motion of the observer.

The linearized measurement matrix is

$$H(X) = \left.\frac{\partial h}{\partial X}\right|_{\tilde{X}} = f/y_s \cdot \begin{bmatrix} 1 & -x_s^2 \end{bmatrix} \cdot T_{w2s} . \quad (7)$$

The dependence of the target motion estimator on the observer motion appears in Equations (4) - (7). It is this dependence that is exploited by the trajectory design method in order to optimize the estimator performance.

Note, in practice the filter implementation is carried out in a transformed coordinate system. The results presented later in this paper use an Extended Kalman Filter (EKF) based on the modified polar coordinates proposed by Aidala and Hammel [9].

3. Trajectory Design

The trajectory design method is comprised of four steps. First, the permissible observer motion is defined in order to set the search space for the optimization. Many formulations are possible [5, 11] and the one presented here keeps the search space small. Second, the constraints that describe the limitations of the system are defined. Third, the cost functions are identified that specify the goals of the optimization. Finally, the optimization method finds the solution that satisfies the observer model, the constraints, and the cost function. The new method described here generates observer trajectories in real-time using an iterative search algorithm.

Observer Motion

The observer path is restricted to a finite number of maneuvers as shown in Figure 2. It has been shown that at least a single maneuver is needed in order to make the target motion observable [4]. The observer speed and maneuver duration are kept constant in order to reduce the complexity of the optimization. The time for each maneuver is distributed between a zero-radius turn

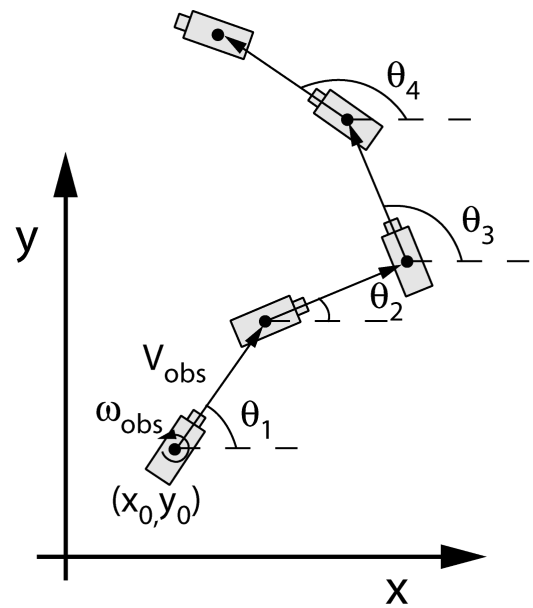

Figure 2. Trajectory parameterization

(T_{turn}) and a constant velocity traverse (T_{leg}) which follows - a longer turn leads to a shorter traverse. The size and number of maneuvers can be adjusted to make the observer path arbitrarily smooth. The free variable is the observer heading for each maneuver.

The equations describing the allowed observer motion for this work are as follows:

$$\begin{aligned} x_{obs}[k+1] &= x_{obs}[k] + VT_{leg}\cos(\theta_{k+1}) \\ y_{obs}[k+1] &= y_{obs}[k] + VT_{leg}\sin(\theta_{k+1}) \end{aligned} \quad (8)$$

$$\begin{aligned} T_{leg} &= T_{man} - T_{turn} \\ T_{man} &= T_{total}/n \\ T_{turn} &= (\theta_{k+1} - \theta_k)/\omega \end{aligned} \quad (9)$$

where θ is the observer heading, V is the observer speed, ω is the observer angular velocity, T_{man} is the duration of each maneuver, T_{total} is the total trajectory duration, and n is the number of maneuvers.

Optimization Problem

Two different optimization problems are formulated using the area of the predicted position uncertainty ellipse as defined by the position estimate error covariance matrix. This ellipse represents a region around the target estimate in which the true target state is located with some confidence. The area is used to describe the ellipse because it can easily be calculated from the determinant of the covariance matrix. Reducing this area is equivalent to reducing the uncertainty of the estimate.

The first optimization problem uses the area of the uncertainty ellipse as the cost function. It is minimized for a fixed number and duration of maneuvers and can be written:

$$\min_{\Theta}(A_{ellipse}(\Theta | T_{man}, n)) . \tag{10}$$

For the second, the number of maneuvers is used as the cost. It is minimized for a fixed maneuver duration and specified final ellipse area:

$$\min_{\Theta}(n) \tag{11}$$

subject to $A_{ellipse}(\Theta, T_{man}, n) \le A_{des}$.

The optimization method below will enable the use of either of these formulations.

The area of the position uncertainty ellipse is related to the target position estimate covariance by the expression

$$A_{ellipse} = \pi \cdot \sqrt{det(P_{cov})} \tag{12}$$

$$\begin{aligned} P_{cov} &= \begin{bmatrix} P_{xx} & P_{xy} \\ P_{yx} & P_{yy} \end{bmatrix} \\ P_{xx} &= E[(x-\tilde{x}) \cdot (x-\tilde{x})] \\ P_{yy} &= E[(y-\tilde{y}) \cdot (y-\tilde{y})] \\ P_{xy} = P_{yx} &= E[(x-\tilde{x}) \cdot (y-\tilde{y})] \end{aligned} \tag{13}$$

The difficulty with the estimate error covariance is predicting its value beforehand. This is overcome by using the target estimate at the time the design method is invoked to predict the target motion. Using this motion, the estimator equations are propagated forward for each candidate trajectory. The resulting predicted error covariances are then used to calculate the costs for the trajectories.

Constraints

The vision field of view and the estimate uncertainty ellipse produce two limitations that must be incorporated into the trajectory design method. A field of view constraint is enforced when propagating the estimator equations, as described above. The estimate is only updated when the predicted target is in view of the candidate trajectory.

A second constraint prevents the observer from entering the uncertainty ellipsoid around the target estimate. Because the target could be anywhere within this ellipsoid with significant probability, it is necessary to include a constraint that keeps the observer out.

This constraint may be represented by

$$\begin{bmatrix} x_{est} - x_{obs} \\ y_{est} - y_{obs} \end{bmatrix}^T \cdot P_{cov} \cdot \begin{bmatrix} x_{est} - x_{obs} \\ y_{est} - y_{obs} \end{bmatrix} \ge c \tag{14}$$

where P_{cov} is the predicted position error covariance from Equation (13) and c is a scaling factor. The constraint is applied to every point along a candidate trajectory.

Optimization Method

The trajectory design problem is solved by performing a pyramid, breadth-first search. For either cost function, a breadth-first search tree is used to generate the list of candidate trajectories that satisfy all the given constraints and to calculate the predicted error covariances that describe those trajectories. The best candidate trajectory is identified and the process is repeated using this trajectory to narrow the search.

Although the ideal result of the optimization is the global minimum, this problem typically cannot be solved in a reasonable amount of time [8]. The search space has too many local minima for gradient-based methods and is too large for an exhaustive search. Instead, a sub-optimal result is obtained in real-time using an iterative pyramid scheme.

The free variables for the trajectory optimization are the headings of each maneuver. The heading space is discretized into a specified number of equal intervals. The number of intervals will determine the size of the search tree and the duration of the optimization process.

The search tree is expanded outward from the initial observer position in a breadth-first fashion. For each possible heading value, the observer motion and the estimator equations are propagated forward in time and the resulting position is checked against the constraints. If the new position satisfies the constraints it is added.

Each node in the search tree describes an observer state and the results of the propagated estimator up to that state. Because there is a unique path from the node back to the root of the tree, each node also represents a trajectory from the original observer state to the node state. Also, because the results of the estimator prediction to that point are stored at the node, the propagation of the estimator to the next node is only a single computation and the complete estimation does not need to be recalculated.

For the fixed-time minimum-uncertainty optimization the search tree is simply expanded to a depth that reaches the desired time. Once this level is reached the trajectory with the lowest cost is the solution.

For the fixed-uncertainty minimum-time optimization the search tree is expanded until the desired uncertainty is reached. Because the expansion is breadth first, the optimal trajectory will be the first one encountered that meets the desired uncertainty.

Once the optimal trajectory is found, the heading space is rediscretized about that result at a finer resolution. The breadth-first search is performed again and the new solution is found. This procedure is repeated a specified number of times until the final desired heading space resolution is achieved. In practice four to six iterations are sufficient to achieve a heading space resolution of less than one degree.

4. Results

This section presents simulation results demonstrating the trajectory design method. The observer motion is based on a rover moving with a constant speed of 0.05 meters per second and a turning rate of 0.5 radians per second. The design method is invoked using the fixed-time minimum-uncertainty cost function. The observer is allowed five, 4.8 second maneuvers. The heading space is discretized into five intervals and the pyramid iteration is performed four times. For this simulation the computer vision system was modelled as a pinhole camera with a field of view of 20 degrees.

The observer begins at the location (-0.86, 0.11) meters and the target begins at (0.73, 0.62) and moves with a constant velocity of (0.0, -0.015) meters per second The design method plans using these values for the target motion. The resulting observer path is shown in Figure 3 along with the true target path and the path of the target estimate over the course of the run The design method took 1.3 seconds to generate the observer trajectory.

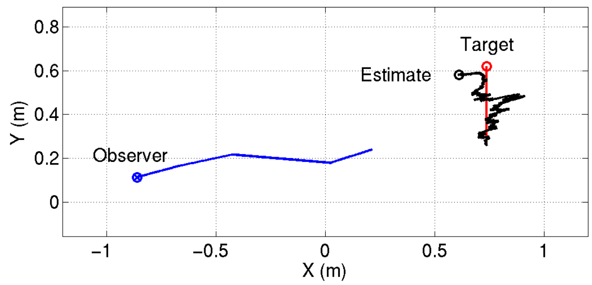

Figure 3. Observer, target, and estimate trajectories

The modified polar coordinate estimator [8] is used to calculate the position and velocity of the target. The trajectory design method is invoked after the estimator has run for several seconds. After this time the target estimate is at (0.61, 0.58) meters with a velocity of (0.04, 0.005) meters per second. Figure 3 shows the observer, target, and estimate curves from the time the design method is called.

There are three main features of the observer path. First, the observer moves toward the target throughout the path. As the range to the target decreases, the achievable bearing rate increases, allowing for wider triangulation and thus smaller error. Second, while moving toward the target, the observer also moves tangential to the line of sight to the target. It is this sideways motion that creates the bearing change needed to observe the target. Finally, the observer turns away from the target at least once. For nonholonomic vehicles, such as airplanes or rovers, this may cause the vision system to lose sight of the target. Procedures need to be implemented in order to reacquire the target should it get lost.

The results of the target motion estimator are shown in Figure 4 and Figure 5. The first figure shows the position error as a function of time. As expected the error in the y-dimension stays small throughout the run. In contrast, the error in the x-dimension, which corresponds closely to the target range, takes time to converge. However, by the end of the path the estimate error is small and the observer has successfully estimated the target motion. The second figure shows the velocity errors. Both components of the velocity converge throughout the run. The large spikes occur when the observer is stopped or turning and are an artifact of the estimator formulation.

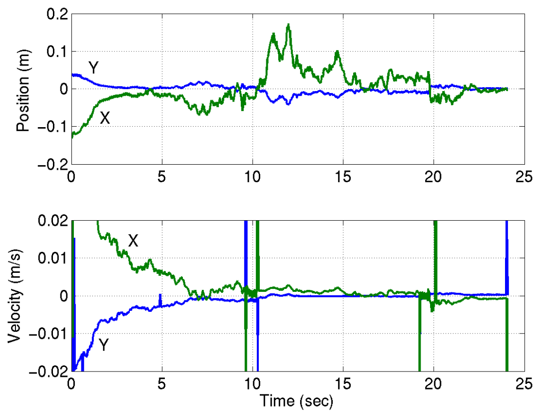

Figure 4. Target estimate errors

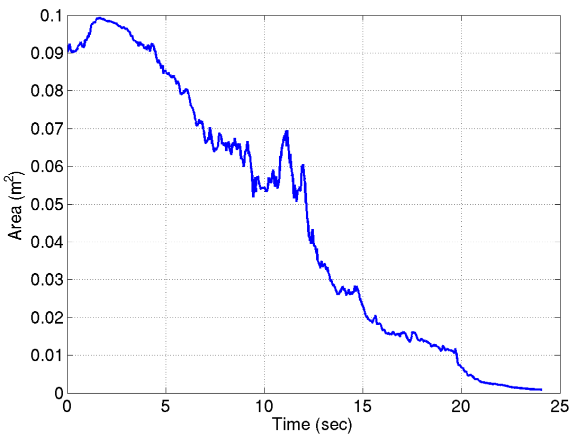

Figure 5. Area of the position uncertainty ellipse

Figure 5 shows a plot of the area of the uncertainty ellipse as a function of time. At the end of the observer trajectory the designer predicted an uncertainty area of 0.0082 square meters. The achieved error was 8.5e-04 square meters. This conservative result is due to the fact that the design method assumes measurements are only made at the end of each maneuver, while the estimator is actually run at 30 Hz.

The results of Figure 4 and Figure 5 show the successful estimation of the target motion. Furthermore, these results show that the predicted error covariance is a good description of the expected estimate performance and that basing the trajectory design method on this description is appropriate.

5. Conclusion

A method was presented for the generation of observer trajectories that improve monocular vision based estimation of a constant velocity target. Estimator performance is optimized by minimizing functions of the predicted target error covariance. In order to generate results in real-time, a sub-optimal iterative breadth-first search algorithm was described. Simulation results showed the success of the trajectory design method for a typical scenario.

6. References

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